Analyzing My Classroom: Acoustic Modeling Using the Finite-Difference Time-Domain Method

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Introduction

Our senses are one of the most vital parts of how we understand our surroundings. One environment that students and faculty at USC spend most of their time in, is in the classroom. While what we see is important, what we don't see, sound, is equally important. Sound waves are always around us, we just don't see them as they propagate. Sound waves can be described by partial differential equations. Consider the wave equation for a sound wave: $\frac{1}{c^2}\partial_t^2 u = \partial_x^2 u + \partial_y^2 u + \partial_z^2 u$ [1], where c is the speed of sound (343 m/s). This mathematical formulation of how the sound wave behaves allows us to compare the observed acoustics to a mathematically formulated acoustical inference. We wanted to see how sound acts in a USC classroom (DMC 157), looking at places it is reflected or absorbed, and how that may affect the educational experience in the room and attempt to replicate the results using the numerical method, Finite-Difference Time-Domain, which would allow us to visualize sound.

Acoustic and Numeric Preliminaries

Sound travels as a wave, with higher frequencies having a shorter wavelength and lower frequency having a longer wavelength. When sound is started at a source, it travels through a medium (the air) and moves through the medium until reaching a boundary, where it is then reflected, dispersed, or absorbed, based on the type of material it is hitting. A test where a sound is played at a point in a space and recorded at another point is known as an impulse response (IR) and allows us to analyze the acoustics of the space through criteria such as reverberance which is the amount of time (in seconds) it takes for an impulse to decrease by 60 dB. Now that we have the proper tools to analyze the acoustics of a room, it is also necessary to understand the methods that can simulate these acoustics. One primary method for solving the wave equation is Finite-Difference Time-Domain. The idea behind FDTD is it discretizes space and time, meaning it breaks them up into small pieces and looks at the pressure value of the scalar field at each of the points. This means that it is volatile to small changes in grid spacing, but smaller grid spacing also means more computational resources. Consider the previous equation, FDTD takes the differences in time, x-direction, y-direction, and z-direction steps and looks at the new values based on the previous ones. For a 2-dimensional case, the finite differences are: $u^n_{l,m} \cong u(n\Delta, l\Delta x, m\Delta y)$ which can be applied to the wave equation:

$$\frac{u^{n+1}_{l,m} - 2u^n_{l,m} + u^{n-1}_{l,m}}{(c\Delta t)^2} = \frac{u^n_{l+1,m} - 2u^n_{l,m} + u^n_{l-1,m}}{(c\Delta x)^2} + \frac{u^n_{l,m+1} - 2u^n_{l,m} + u^n_{l,m-1}}{(c\Delta y)^2}$$

note that this is for 2-dimensions [2]. For 3-dimensions, we include the z term. By splitting up space and time into a grid, we can understand the behavior of a sound wave at each point in space and time using numerical methods.

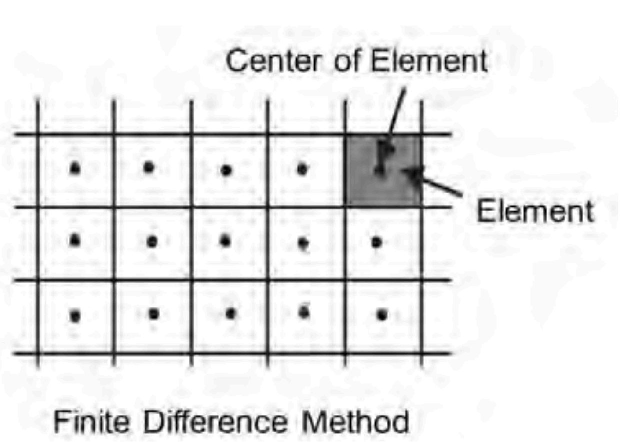

Figure 1. Finite-Difference Time-Domain Visual [3]

Methodology

In order to accurately simulate the acousitcs of the room, we first needed to measure the physical properties of the room. Using a tape measure, we recorded the dimensions of the space to be: 7.12m x 6.02m x 2.72m. Next, we created a 2-dimensional and 3-dimensional floor plan of the space using a LiDAR scanner, which generated the following:

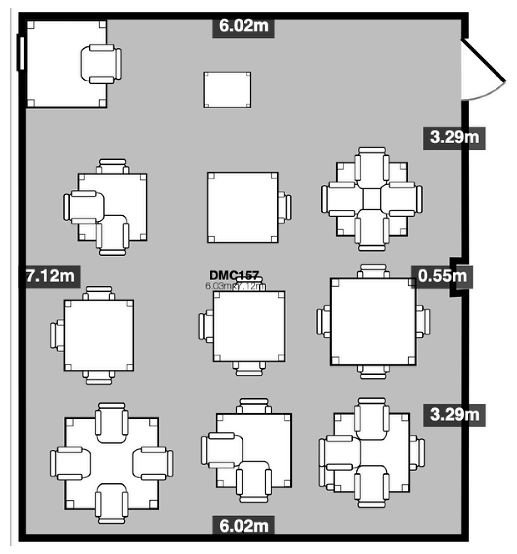

Figure 2. 2D Floor Plan for DMC157 [4]

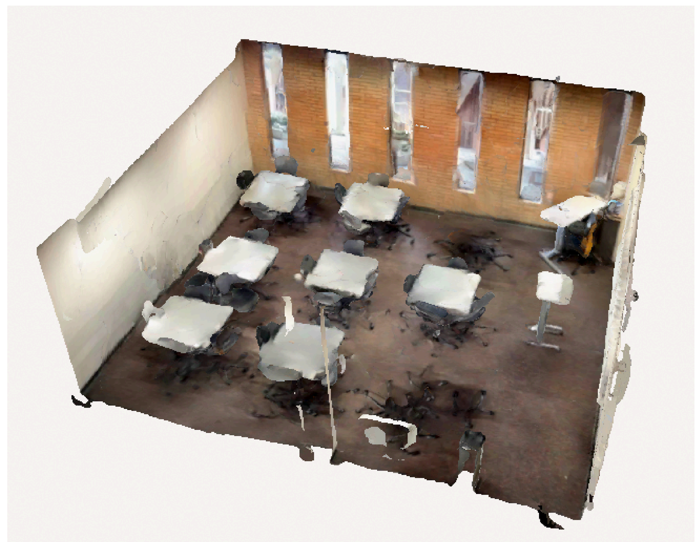

Figure 3. 3-dimensional LiDAR scan of DMC157 [5]

To analyze the classroom accurately, we noted the materials of the room. One wall was brick with windows and the other three were concrete. Concrete and glass for windows are very reflective materials. The next phase consisted of recording impulse responses (IR), so we could analyze the acoustics of the space, and see if we could replicate the results. For the IR, we chose a tone that goes through the range of human hearing and recorded it at several locations in the classroom. The positions we chose were typical positions that students and professors would be speaking or hearing from. One iPhone was used to record the impulse and another to play it.

Results

Using REW, we were able to generate the impulse response graphs, along with reverberation times.

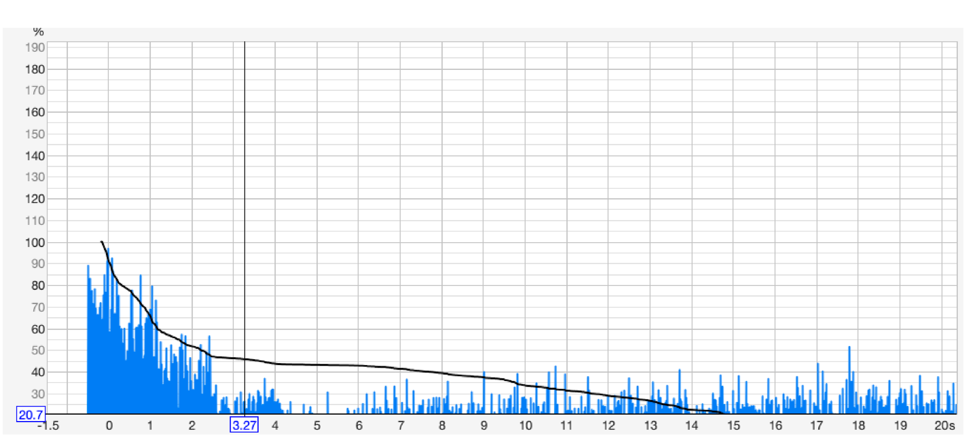

Figure 4. Recorded Impulse Response for Front-Middle of DMC 157

Table 1
Reverberation times at different frequencies for an impulse response at the front middle of DMC 157

Frequency (Hz)	Reverberation Time (Seconds)
50	21.11
80	14.13
100	21.96
200	23.89
300	5.89
1,200	24.56
1,600	23.30
3,000	19.94
5,000	4.12
6,000	2.52
8,000	3.45

By acoustically analyzing DMC 157 we found that there are extremely high reverberation times, specifically at speaking frequencies (80-300 Hz). Our observed reverberation times ranged from 14-23 seconds at speaking range, where the ideal reverberation time for a small classroom should be 0.6-0.7 seconds [6]. This means that the materials in DMC 157 are very reflective, causing copies of the initial sound wave to propagate, causing the initial sound to emanate for longer. This could have negative effects on students in the classroom due to high reverberation times causing distractions.

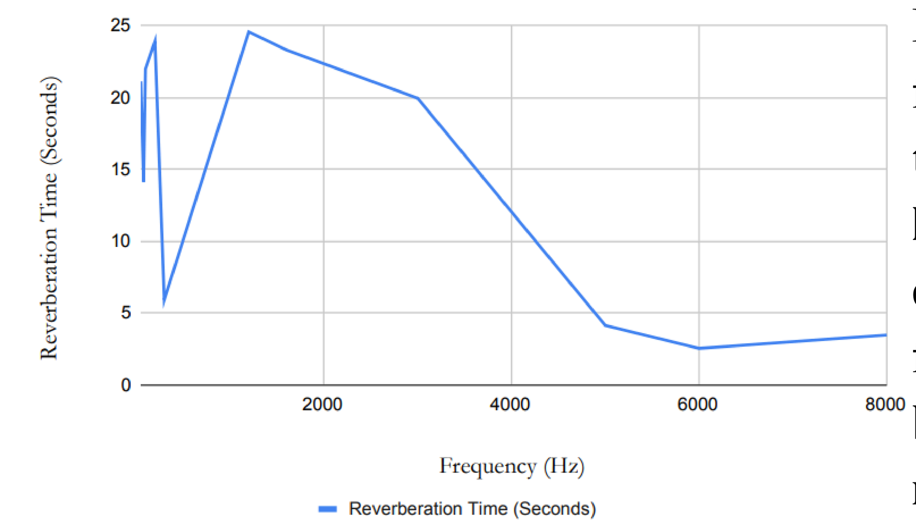

Figure 5. Reverberation/frequency plot for IR in DMC 157 (Front-Middle)

From Figure 5, it is clear that as frequency decreases, the reverberation time also decreases. This is due to higher frequencies being more directional which leads to the lower frequencies having a higher chance of being reflected, causing higher reverberation times.

Using numerical methods and FDTD, we also ran a simulation of the room in order to see if we could generate similar results using mathematics. The 3-dimensional LiDAR scan was used in our model and the scalar pressure field was calculated at each grid point in x,y,z at each time step, t [7].

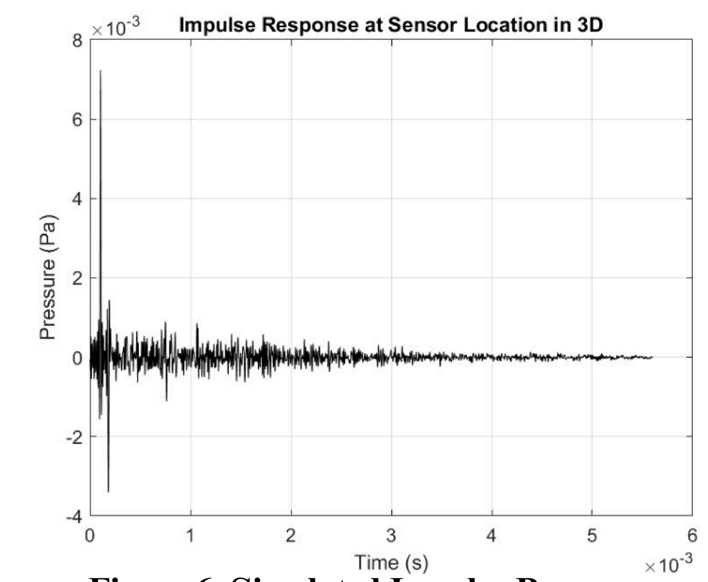

Figure 6. Simulated Impulse Response

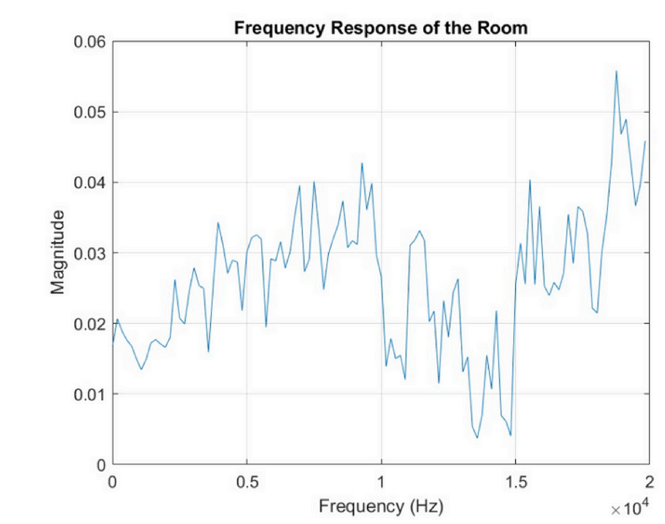

Figure 7. Magnitude/Frequency for Simulation

The simulation confirmed the general trend that the reverberation time was higher at lower frequencies, with a slight difference where the reverberation time increases again at higher frequencies. However, due to the amount of calculations necessary, the simulation was only able to generate a short impulse response. Both the observed data and the simulated data confirmed that due to the abundance of reflective materials that make up DMC 157, there is a very high reverberation time that causes many reflections and can have negative effects on education at USC.

Conclusion

Acoustics are often overlooked, but play a vital role in a learning environment, where clarity between student and professor is very important. By analyzing impulse responses and reverberation times, we can better understand the role acoustics play and make recommendations about how to optimize acoustics. Additionally, simulations can prove useful for being able to handle many conditions without using physical resources. However, due to the computational intensity of FDTD, it is difficult to run longer simulations. A further research direction would be determining a way to optimize FDTD for computational ease.

References and Acknowledgements

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